



# Standard Test Method for Young's Modulus, Tangent Modulus, and Chord Modulus<sup>1</sup>

This standard is issued under the fixed designation E111; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon ( $\epsilon$ ) indicates an editorial change since the last revision or reapproval.

*This standard has been approved for use by agencies of the U.S. Department of Defense.*

## 1. Scope

1.1 This test method<sup>2</sup> covers the determination of Young's modulus, tangent modulus, and chord modulus of structural materials. This test method is limited to materials in which and to temperatures and stresses at which creep is negligible compared to the strain produced immediately upon loading and to elastic behavior.

1.2 Because of experimental problems associated with the establishment of the origin of the stress-strain curve described in 8.1, the determination of the initial tangent modulus (that is, the slope of the stress-strain curve at the origin) and the secant modulus are outside the scope of this test method.

1.3 The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.4 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory requirements prior to use.*

## 2. Referenced Documents

### 2.1 ASTM Standards:<sup>3</sup>

- E4 Practices for Force Verification of Testing Machines
- E6 Terminology Relating to Methods of Mechanical Testing
- E8 Test Methods for Tension Testing of Metallic Materials
- E9 Test Methods of Compression Testing of Metallic Materials at Room Temperature
- E21 Test Methods for Elevated Temperature Tension Tests of Metallic Materials
- E83 Practice for Verification and Classification of Extensometer Systems
- E231 Method for Static Determination of Young's Modulus of Metals at Low and Elevated Temperatures (Withdrawn 1985)<sup>4</sup>
- E1012 Practice for Verification of Testing Frame and Specimen Alignment Under Tensile and Compressive Axial Force Application

<sup>1</sup> This test method is under the jurisdiction of ASTM Committee E28 on Mechanical Testing and is the direct responsibility of Subcommittee E28.04 on Uniaxial Testing.

Current edition approved Sept. 15, 2010. Published January 2011. Originally approved in 1955. Last previous edition approved in 2004 as E111 – 04. DOI: 10.1520/E0111-04R10

<sup>2</sup> This test method is a revision of E111 – 61 (1978), "Young's Modulus at Room Temperature" and includes appropriate requirements of E231 – 69 (1975), "Static Determination of Young's Modulus of Metals at Low and Elevated Temperatures" to permit the eventual withdrawal of the latter method. Method E231 is under the jurisdiction of ASTM-ASME Joint Committee on Effect of Temperature on the Property of Metals.

<sup>3</sup> For referenced ASTM standards, visit the ASTM website, [www.astm.org](http://www.astm.org), or contact ASTM Customer Service at [service@astm.org](mailto:service@astm.org). For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

Materials at Room Temperature

E21 Test Methods for Elevated Temperature Tension Tests of Metallic Materials

E83 Practice for Verification and Classification of Extensometer Systems

E231 Method for Static Determination of Young's Modulus of Metals at Low and Elevated Temperatures (Withdrawn 1985)<sup>4</sup>

E1012 Practice for Verification of Testing Frame and Specimen Alignment Under Tensile and Compressive Axial Force Application

2.2 *General Considerations*—While certain portions of the standards and practices listed are applicable and should be referred to, the precision required in this test method is higher than that required in general testing.

## 3. Terminology

### 3.1 Definitions:

3.1.1 *accuracy*—the degree of agreement between an accepted standard value of Young's modulus (the average of many observations made according to this method, preferably by many observers) and the value determined.

3.1.1.1 Increased accuracy is associated with decreased bias relative to the accepted standard value; two methods with equal bias relative to the accepted standard value have equal accuracy even if one method is more precise than the other. See also *bias* and *precision*.

3.1.1.2 The accepted standard value is the value of Young's modulus for the statistical universe being sampled using this method. When an accepted standard value is not available, accuracy cannot be established.

3.1.2 *bias, statistical*—a constant or systematic error in test results.

3.1.2.1 Bias can exist between the accepted standard value and a test result obtained from this test method, or between two test results obtained from this test method, for example, between operators or between laboratories.

<sup>4</sup> The last approved version of this historical standard is referenced on [www.astm.org](http://www.astm.org).

3.1.3 *precision*—the degree of mutual agreement among individual measurements made under prescribed like conditions.

3.1.4 *Young’s modulus*—the ratio of tensile or compressive stress to corresponding strain below the proportional limit (see Fig. 1a).

3.1.4.1 *tangent modulus*—the slope of the stress-strain curve at any specified stress or strain (see Fig. 1b).

3.1.4.2 *chord modulus*—the slope of the chord drawn between any two specified points on the stress-strain curve (see Fig. 1c).

3.2 For definitions of other terms used in this test method, refer to Terminology E6.

**4. Summary of Test Method**

4.1 A uniaxial force is applied to the test specimen and the force and strain are measured, either incrementally or continuously. The axial stress is determined by dividing the indicated force by the specimen’s original cross-sectional area. The appropriate slope is then calculated from the stress-strain curve, which may be derived under conditions of either increasing or decreasing forces (increasing from preload to maximum applied force or decreasing from maximum applied force to preload).

**5. Significance and Use**

5.1 The value of Young’s modulus is a material property useful in design for calculating compliance of structural materials that follow Hooke’s law when subjected to uniaxial loading (that is, the strain is proportional to the applied force).

5.2 For materials that follow nonlinear elastic stress-strain behavior, the value of tangent or chord modulus is useful in estimating the change in strain for a specified range in stress.

5.3 Since for many materials, Young’s modulus in tension is different from Young’s modulus in compression, it shall be derived from test data obtained in the stress mode of interest.

5.4 The accuracy and precision of apparatus, test specimens, and procedural steps should be such as to conform to the material being tested and to a reference standard, if available.

5.5 Precise determination of Young’s modulus requires due regard for the numerous variables that may affect such determinations. These include (1) characteristics of the specimen such as orientation of grains relative to the direction of the stress, grain size, residual stress, previous strain history, dimensions, and eccentricity; (2) testing conditions, such as alignment of the specimen, speed of testing, temperature, temperature variations, condition of test equipment, ratio of

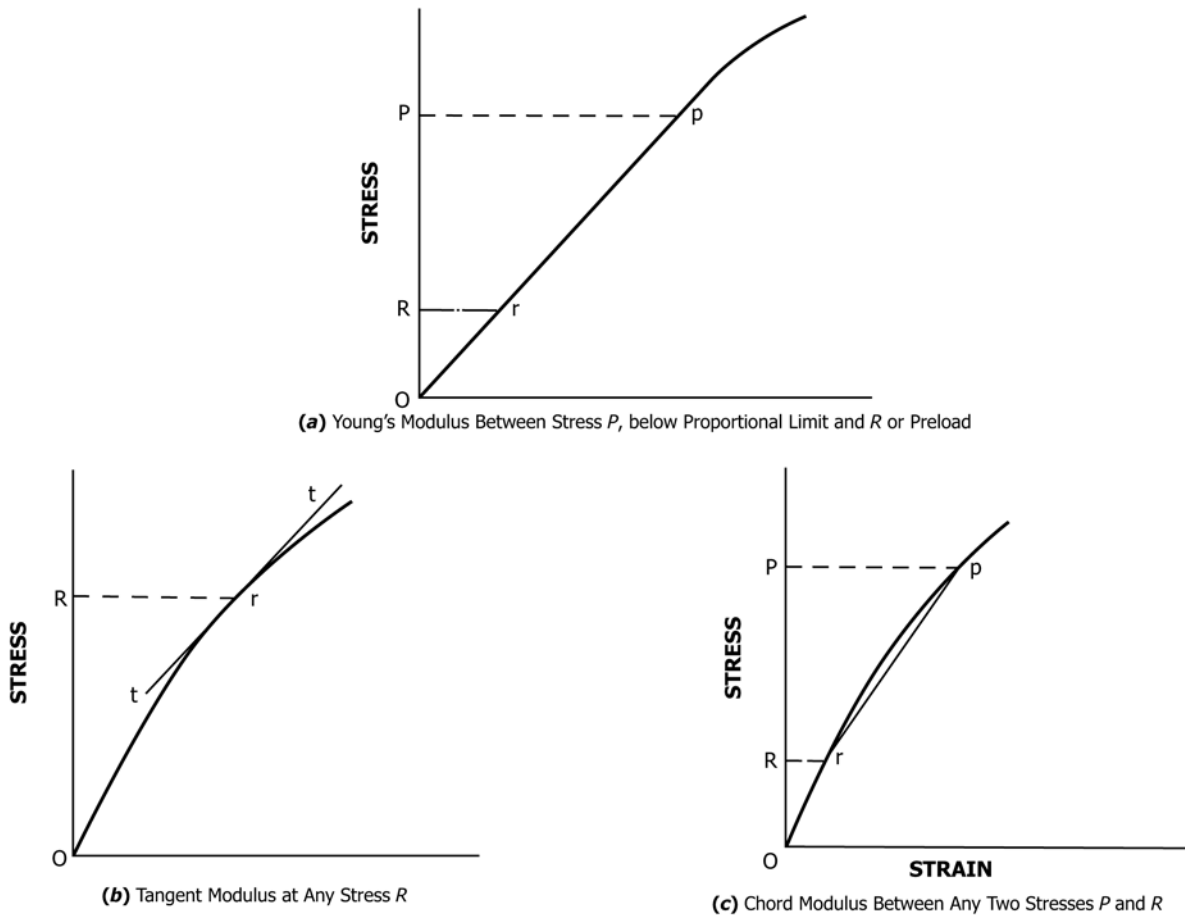


FIG. 1 Stress-Strain Diagrams Showing Straight Lines Corresponding to (a) Young’s Modulus, (b) Tangent Modulus, and (c) Chord Modulus

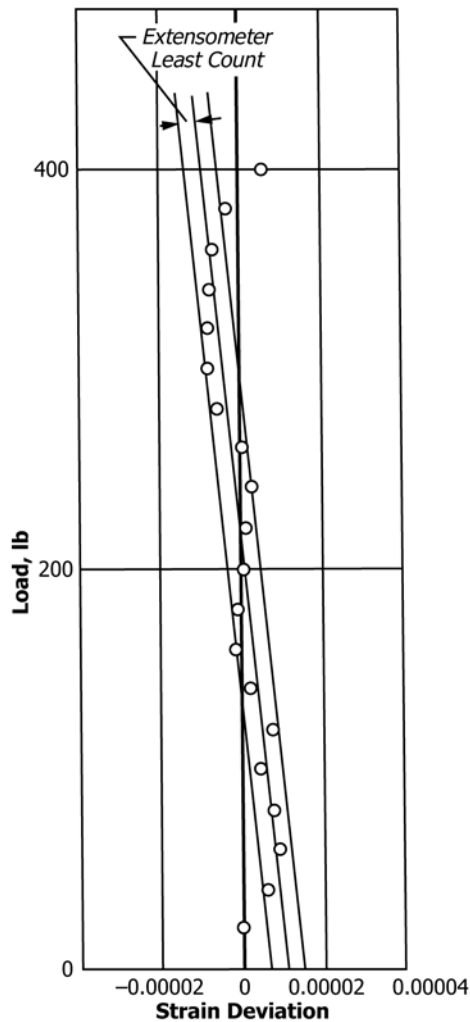


FIG. 2 Load-Deviation Graph

error in applied force to the range in force values, and ratio of error in extension measurement to the range in extension values used in the determination; and (3) interpretation of data (see Section 9).

5.6 When the modulus determination is made at strains in excess of 0.25 %, correction should be made for changes in cross-sectional area and gage length, by substituting the instantaneous cross section and instantaneous gage length for the original values.

5.7 Compression results may be affected by barreling (see Test Methods E9). Strain measurements should therefore be made in the specimen region where such effects are minimal.

## 6. Apparatus

6.1 *Dead Weights*—Calibrated dead weights may be used. Any cumulative errors in the dead weights or the dead weight loading system shall not exceed 0.1 %.

6.2 *Testing Machines*—In determining the suitability of a testing machine, the machine shall be calibrated under conditions approximating those under which the determination is made. Corrections may be applied to correct for proven systematic errors.

6.3 *Loading Fixtures*—Grips and other devices for obtaining and maintaining axial alignment are shown in Test Methods E8 and E9. It is essential that the loading fixtures be properly designed and maintained. Procedures for verifying the alignment are described in detail in Practice E1012. The allowable bending as defined in Practice E1012 shall not exceed 5 %.

6.4 *Extensometers*—Class B-1 or better extensometers as described in Practice E83 shall be used. Corrections may be applied for proven systematic errors in strain and are not considered as a change in class of the extensometer. Either an averaging extensometer or the average of the strain measured by at least two extensometers arranged at equal intervals around the cross section be used. If two extensometers are used on other than round sections, they shall be mounted at ends of an axis of symmetry of the section. If a force-strain recorder, strain-transfer device, or strain follower is used with the extensometer, they shall be calibrated as a unit in the same manner in which they are used for determination of Young’s modulus. The gage length shall be determined with an accuracy consistent with the precision expected from the modulus determination and from the extensometer.

NOTE 1—The accuracy of the modulus determination depends on the precision of the strain measurement. The latter can be improved by increasing the gage length. This may, however, present problems in maintaining specimen tolerances and temperature uniformity.

6.5 *Furnaces or Heating Devices*—When determining Young’s modulus at elevated temperature, the furnace or heating device used shall be capable of maintaining a uniform temperature in the reduced section of the test specimen so that a variation of not more than  $\pm 1.5^{\circ}\text{C}$  for temperatures up to and including  $900^{\circ}\text{C}$ , and not more than  $\pm 3.0^{\circ}\text{C}$  for temperatures above  $900^{\circ}\text{C}$ , occurs. (Heating by self-resistance is not accepted.) Minimize temperature variations and control changes within the allowable limits, since differences in thermal expansion between specimen and extensometer parts may cause significant errors in apparent strain. An instrumented sample representative of the real test will demonstrate that the setup meets the above capabilities.

6.6 *Low-Temperature Baths and Refrigeration Equipment*—When determining Young’s modulus at low temperatures, an appropriate low-temperature bath or refrigeration system is required to maintain the specimen at the specified temperature during testing. For a low-temperature bath, the lower tension rod or adapter may pass through the bottom of an insulated container and be welded or fastened to it to prevent leakage. For temperatures to about  $-80^{\circ}\text{C}$ , chipped dry ice may be used to cool an organic solvent such as ethyl alcohol in the low-temperature bath. Other organic solvents having lower solidification temperatures, such as methylcyclohexane or isopentane, may be cooled with liquid nitrogen to temperatures lower than  $-80^{\circ}\text{C}$ . Liquid nitrogen may be used to achieve a testing temperature of  $-196^{\circ}\text{C}$ . Lower testing temperatures may be achieved with liquid hydrogen and liquid helium, but special containers or cryostats are required to provide for minimum heat leakage to permit efficient use of these coolants. When liquid hydrogen is used, special precautions must be taken to avoid explosions of hydrogen gas and air mixtures. If refrigeration equipment is used to cool the specimens with air